Abstract – Active noise control using TSK Fuzzy algorithm is proposed to contend a nonlinearity problem in primary and secondary path, where standard FxLMS algorithms may not perform well in cases where nonlinearities are found. However, there may be some uncertainties in fuzzy systems. Type-1 fuzzy system, which has crip membership grade, may not be able to handle this uncertainties. Interval type-2 fuzzy system, which has interval membership grade, is powerful in handling this uncertainties. In this paper, an adaptive type-2 TSK fuzzy system is developed for modeling nonlinear model of ANC secondary path and for ANC nonlinear controller. Performance of interval type-2 TSK fuzzy system for ANC is compared with type-1 TSK Fuzzy and FxLMS algorithm. The results show that interval type-2 TSK fuzzy systems perform better than type-1 TSK fuzzy system and FxLMS algorithms in both modeling and control of active noise control.

Index Terms - Active noise control, interval type 2 TSK adaptive Fuzzy system, learning algorithm, nonlinearity

I. INTRODUCTION

Active Noise Control (ANC) is a method for attenuating unwanted disturbances of acoustic noise based on the principle of destructive interferences between an original “primary” disturbance field and a “secondary” field that is generated by some control actuators [1]. A fundamental problem to be considered in ANC
systems is the requirement of highly precise control, temporal stability and reliability. In order to produce a high degree of attenuation, the amplitude and phase of both the primary and the secondary noises must match with close precision. The filtered-x LMS (FxLMS) algorithm is the most commonly used algorithm in ANC because of its simple implementation. In the FxLMS algorithm shown in Fig. 1, the main path $P(z)$ defines the path from the noise source to the cancellation point $e(n)$. The secondary path $S(z)$ defines the path leading from the adaptive filter output to the cancellation point. The secondary path $S(z)$ usually contains a D/A converter, an amplifier and an actuator. In ANC systems, the actuator is most often an audio loudspeaker.

![Block diagram of a feedforward active control system using FxLMS algorithm](image)

Fig. 1. Block diagram of a feedforward active control system using FxLMS algorithm

Typically in ANC both main and secondary path are assumed be linear and it is further assumed that linear filtering will effectively deal with the sound signals. However, in actual systems, the plant may be nonlinear, the reference noise may exhibit nonlinear distortion and the secondary path may also have some nonlinear distortion. The linear digital control may not perform well in cases where nonlinearities are found in an active control system [2].

Various types of nonlinear control have been studied in nonlinear ANC, such as truncated Volterra expansions [3], multi-layer artificial neural network or fuzzy neural network [2]. This paper explores nonlinear active control systems using interval type-2 TSK adaptive Fuzzy system (T2TSK), which is recently shown to possess capability in handling uncertainties [8]. Motivated by the ANC structure in [8]
using neural networks, two T2TSK adaptive structures are employed in ANC: the first structure is used to model ANC secondary path, while the second one serves as adaptive nonlinear controller. In the sequel, the former is called Model T2TSK, while the latter is referred to as Control T2TSK. The Control T2TSK generates control signal from reference signal, which is then applied to the plant through actuators. The interval type-2 TSK Fuzzy algorithm was implemented in active noise control for single channel case. Performance of interval type 2 TSK fuzzy then comparing between type 1 TSK Fuzzy and FIR algorithm. The comparison between this three algorithm was done by measured MSE, convergence velocity and attenuation level not only for the main frequency but also the harmonic frequency. Although there are recently numerous studies on type-2 Fuzzy systems as discussed in [4-6], as far as the author concerns, there is no work on development of type-2 TSK Fuzzy that is suitable for specific structure of ANC. Contribution of this paper is to propose T2TSK algorithm suitable for active noise control and compare the performance of interval type 2 TSK Fuzzy between type-1 TSK Fuzzy and FIR algorithm.

The paper is arranged as follows. Interval type-2 TSK Fuzzy structure is provided in Section II. Interval type-2 TSK Fuzzy algorithm suitable for active noise control is presented in Section III. Interval type-2 TSK Fuzzy algorithm for identification structure is developed in Section IV. The simulation results are provided in Section V. Conclusion is drawn in in Section VI.
II. T2TSK STRUCTURE

The most popular fuzzy logic systems used by engineers today are Mamdani and TSK systems. Both are characterized by IF-THEN rules and have the same antecedent structures. They differ in the structure of the consequents. The consequent of a Mamdani rule is a fuzzy set, whereas the consequent of a TSK rule is a crisp function [12].

Type-2 fuzzy set was originally presented by Zadeh in 1975. The new concepts were introduced by Mendel and Liang, which allows characterization of a type-2 fuzzy set with a superior membership function and an inferior membership function; these two functions can be represented each one by a type-1 fuzzy set membership function. The interval between these two functions represent the footprint of uncertainty (FOU), which is used to characterize a type-2 fuzzy set. Type-2 fuzzy sets allow us to handle linguistic uncertainties, as typified by the adage “words can mean different things to different people”.

For type-2 TSK models, there are three possible structures [12]:
1. Antecedents are type-2 fuzzy sets, and consequents are type-1 fuzzy sets. This is the most general case and we call it Model I.
2. Antecedents are type-2 fuzzy sets, and consequents are crisp number. This is special case of Model I and we call it model II.
3. Antecedents are type-1 Fuzzy sets and consequents are type-1 fuzzy sets. This is another special case of Model I and we call it Model III.

We use Model I to design interval type-2 TSK Fuzzy system in this paper. A schematic diagram of the proposed T2TSK structure is shown in Fig. 2, which is organized into $i$ input variables and $m$ rules.

In first-order type-2 TSK Model I, a rule base consisting of $m$ rules and $n$ input variables, is denoted as

\[
\text{IF } x_1 \text{ is } \mu_1^b(x_1) \text{ AND ... AND } x_a \text{ is } \mu_a^b(x_a) \\
\text{THEN } Z \text{ is } p_1^b x_1 + p_2^b x_2 + \ldots + p_a^b x_a + p_0^b \tag{1}
\]

where $b \in [0, m]$ and $a \in [0, n]$. The consequent parameters $p_1^b, p_2^b, \ldots, p_a^b, p_0^b$, which are type-1 fuzzy sets, has interval denoted as

\[
p_a^b = [c_a^b - s_a^b, c_a^b + s_a^b] \tag{2}
\]

The membership grades $\mu_1^b(x_1), \mu_2^b(x_2), \ldots, \mu_a^b(x_a)$ are interval sets denoted as
\[ \mu_a^b = \left[ \mu_a^b, \bar{\mu}_a^b \right] \]  

(3)

where \( \mu_a^b \) is lower membership function and \( \bar{\mu}_a^b \) is upper membership function.

The primary membership functions for each antecedent are interval type-2 fuzzy systems described by Gaussian primary membership function with uncertain means, denoted as

\[ \mu_a^b(x_a) = \exp \left[ -\frac{1}{2} \left( \frac{x_a - m_a^b}{\sigma_a^b} \right)^2 \right] \]  

(4)

where \( m_a^b \in [m_{a1}^b, m_{a2}^b] \) is the uncertain mean, \( a = (1, \ldots, n) \) is the number of antecedent, \( b = (1, \ldots, m) \) is the number of rules and \( \sigma_a^b \) is the standard deviation.

There are two kinds of type-2 sets. First is a gaussian type-2 fuzzy set, in which the membership grade of every domain point is a Gaussian type-1 set contained in [0,1]. The other type is an interval type-2 fuzzy set in which the membership grade of every domain point is a crisp set whose domain is some interval contained in [0,1]. Fig 3 shows gaussian interval type-2 fuzzy membership function with uncertain means.

The upper membership function is defined as

\[ \bar{\mu}_a^b(x_a) = \begin{cases} N(m_{a1}^b, \sigma_a^b, x_a), & x_a < m_{a1}^b \\ 1, & m_{a1}^b \leq x_a \leq m_{a2}^b \\ N(m_{a1}^b, \sigma_a^b, x_a), & x_a > m_{a2}^b \end{cases} \]  

(5)

where

\[ N(m_{a1}^b, \sigma_a^b, x_a) = \exp \left[ -\frac{1}{2} \left( \frac{x_a - m_{a1}^b}{\sigma_a^b} \right)^2 \right] \]  

(6)

The lower membership function is defined as

\[ \mu_a^b(x_a) = \begin{cases} N(m_{a1}^b, \sigma_a^b, x_a), & x_a \leq \frac{m_{a1}^b + m_{a2}^b}{2} \\ N(m_{a2}^b, \sigma_a^b, x_a), & x_a > \frac{m_{a1}^b + m_{a2}^b}{2} \end{cases} \]  

(7)
Fuzzy inference in antecedent utilizes algebraic product, denoted as

\[ W^b = \mu_1^b(x_1) \times \mu_2^b(x_2) \times \cdots \times \mu_n^b(x_n) \] (8)

and

\[ \overline{W}^b = \overline{\mu}_1^b(x_1) \times \overline{\mu}_2^b(x_2) \times \cdots \times \overline{\mu}_n^b(x_n) \] (9)

The interval value of the consequent \( Z^b \) is \( Z^b = [Z_{l}^b, Z_{r}^b] \), where

\[
Z_{l}^b = \sum_{i=1}^{n} c_i^b x_i + c_0^b - \sum_{i=1}^{n} s_i^b |x_i| - s_0^b
\]

\[
Z_{r}^b = \sum_{i=1}^{n} c_i^b x_i + c_0^b + \sum_{i=1}^{n} s_i^b |x_i| + s_0^b
\] (10)

and \( Z_{l}^b \) and \( Z_{r}^b \) denote the lower and upper values of consequent output of the \( b \) th rule.

The Karnik-Mendel algorithms is used for determining \( c_l \) and \( c_r \). The five steps for determining \( c_r \) are as follows [14]:

[1] Initialize \( \theta_r^b \) by setting:

\[
\theta_r^b = \frac{1}{2} \left[ W^b + \overline{W}^b \right] \quad b = 1, \ldots, n
\] (11)

Fig. 3. Gaussian interval type-2 fuzzy membership function with uncertain means
and compute:

\[ c' = \frac{\sum_{b=1}^{m} \theta_r^b Z_r^b}{\sum_{b=1}^{m} \theta_r^b} \]  \hspace{1cm} (12)

[2] Find \( k_r \) \((1 \leq k_r \leq N - 1)\) such that

\[ Z_r^{k_r} \leq c' \leq Z_r^{k_r+1} \]  \hspace{1cm} (13)

[3] Set

\[ \theta_r^b = \begin{cases} \frac{W^b}{b} & b \leq k_r \\ \frac{W^b}{b} & b \geq k_r + 1 \end{cases} \]  \hspace{1cm} (14)

Compute:

\[ c'' = \frac{\sum_{b=1}^{k} Z_r^b W^b + \sum_{b=k+1}^{m} Z_r^b W^b}{\sum_{b=1}^{k} W^b + \sum_{b=k+1}^{m} W^b} \]  \hspace{1cm} (15)

[4] Check if \( c'' = c' \). If yes, stop and set \( c''' = c_r \). If no, go to step [5]

[5] Set \( c' = c'' \) and go to step [2]

Calculation of \( c_l \) is carried-out using the same procedure as above, except that in step 3, we set

\[ \theta_l^b = \begin{cases} \frac{W^b}{b} & b \leq k_l \\ \frac{W^b}{b} & b \geq k_l + 1 \end{cases} \]  \hspace{1cm} (16)

and

\[ c'' = \frac{\sum_{b=1}^{k} Z_l^b W^b + \sum_{b=k+1}^{m} Z_l^b W^b}{\sum_{b=1}^{k} W^b + \sum_{b=k+1}^{m} W^b} \]  \hspace{1cm} (17)

The output of fuzzy system is calculated by
III. T2TSK ALGORITHM FOR ANC

Block diagram of the interval type-2 TSK fuzzy system in active noise control system is shown in Fig. 4. Objective of the active noise control is to generate controlled signal (referred to as anti-noise) from the measurements of the noise source and of the error sensor such that the residual around the error sensor is minimized. As shown in this figure, the reference signal $x(k)$ captured by reference sensor (reference microphone) is not influenced by feedback from secondary source (output of canceling loudspeaker). Both primary path and control T2TSK are driven by the same reference signal. The output of primary path is the primary noise signal $d(n)$. The primary noise signal is to Control T2TSK, used to generate anti noise signal through canceling loudspeaker. The residual noise signal $e(n)$ is captured by error sensor (error microphone) placed in the vicinity of the canceling loudspeaker. This error signal will be minimized when parameters of controller is adapted through on-line training process. T2TSK control structure consists of nonlinear controller (Control T2TSK Fuzzy System) and secondary path model (Model T2TSK Fuzzy System). Before the control system is activated as shown in Fig 2, the weight of secondary path model T2TSK are obtained through off-line identification process.

![Block Diagram of T2TSK Fuzzy System for Active Noise Control](image)

The calculation to weight update from controller T2TSK Fuzzy is expressed as

$$W_c(k + 1) = W_c(k) - \eta \frac{\partial E(k)}{\partial W_c(k)}$$  \hspace{1cm} (19)
Using \( E(k) = \frac{1}{2} e(k)^2 \), it follows that

\[
W_c(k + 1) = W_c(k) - \eta \cdot e \cdot \frac{\partial s(k)}{\partial W_c(k)}
\]

(20)

Using fuzzy output model \( y(k) \) to predict \( s(k) \), we can get

\[
W_c(k + 1) = W_c(k) - \eta \cdot e \cdot \frac{\partial y(k)}{\partial W_c(k)}
\]

(21)

Because we are using Karnik-Mendel algorithm in T2TSK structure, \( \frac{\partial y(k)}{\partial W_c(k)} \) value can we get using following equations.

1. If \( b \geq k_r + 1 \) and \( x_a(k) < m_{a1}^b \)

\[
\frac{\partial y(k)}{\partial W_c(k)} = \sum_k \sum_j \left[ \left( \frac{x_a(k) - m_{j1}^k}{(\sigma_j^k)^2} \right) \cdot \theta_r^j \cdot \frac{\partial y_c^k}{\partial W_c} \right] \cdot \sum_j \theta_r^j z_r^j - \frac{\left( \sum_j \theta_r^j \right)^2}{(\sum_j \theta_r^j)^2}
\]

2. If \( b \geq k_r + 1 \) and \( x_a(k) > m_{a2}^b \)

\[
\frac{\partial y(k)}{\partial W_c(k)} = \sum_k \sum_j \left[ \left( \frac{x_a(k) - m_{j2}^k}{(\sigma_j^k)^2} \right) \cdot \theta_r^j \cdot \frac{\partial y_c^k}{\partial W_c} \right] \cdot \sum_j \theta_r^j z_r^j - \frac{\left( \sum_j \theta_r^j \right)^2}{(\sum_j \theta_r^j)^2}
\]
\[
\sum_k \sum_j \left[ \frac{x_a(k) - m_{j1}^k}{(\sigma_j^k)^2} \cdot \theta_r^j \cdot \frac{\partial y_c^k}{\partial \theta_r^j} \right] - \sum_{j} \theta_r^j \sum_{C_i} c_i \frac{\partial y_c^i}{\partial \theta_r^j} \left( \sum_m \theta_r^j \right)
\]

3. If \( b \leq k_r \) and \( x_a(k) < \frac{m_{a1}^b + m_{a2}^b}{2} \)

\[
\frac{\partial y(k)}{\partial W_c(k)} = \sum_k \sum_j \left[ \frac{x_a(k) - m_{j2}^k}{(\sigma_j^k)^2} \cdot \theta_r^j \cdot \frac{\partial y_c^k}{\partial \theta_r^j} \right] \cdot \sum_{j} \theta_r^j \sum_{l} c_l \frac{\partial y_c^l}{\partial \theta_r^j} \left( \sum_m \theta_r^j \right)
\]

(24)

4. If \( b \leq k_r \) and \( x_a(k) \geq \frac{m_{a1}^b + m_{a2}^b}{2} \)

\[
\frac{\partial y(k)}{\partial W_c(k)} = \sum_k \sum_j \left[ \frac{x_a(k) - m_{j1}^k}{(\sigma_j^k)^2} \cdot \theta_r^j \cdot \frac{\partial y_c^k}{\partial \theta_r^j} \right] \cdot \sum_{j} \theta_r^j \sum_{l} c_l \frac{\partial y_c^l}{\partial \theta_r^j} \left( \sum_m \theta_r^j \right)
\]

(25)

5. If \( b \geq k_l + 1 \) and \( x_a(k) < \frac{m_{a1}^b + m_{a2}^b}{2} \)

\[
\frac{\partial y(k)}{\partial W_c(k)} = \sum_k \sum_j \left[ \frac{x_a(k) - m_{j1}^k}{(\sigma_j^k)^2} \cdot \theta_r^j \cdot \frac{\partial y_c^k}{\partial \theta_r^j} \right] - \sum_{j} \theta_r^j \sum_{l} c_l \frac{\partial y_c^l}{\partial \theta_r^j} \left( \sum_m \theta_r^j \right)
\]

(26)
$$\sum_{k} \sum_{j} \left[ \frac{\left( x_a(k) - m_{j_2}^k \right)}{\left( \sigma_j^k \right)^2} \cdot \theta_j \cdot \frac{\partial y_c^k}{\partial W_c} \right] \cdot \sum_{j} \theta_j^i \cdot z_i^j \cdot \frac{1}{\left( \sum_{j} \theta_j^l \right)^2}$$

$$= \sum_{k} \sum_{j} \left[ \frac{\left( x_a(k) - m_{j_1}^k \right)}{\left( \sigma_j^k \right)^2} \cdot \theta_j \cdot \frac{\partial y_c^k}{\partial W_c} \right] \cdot \sum_{j} \theta_j^i \cdot z_i^j \cdot \frac{1}{\left( \sum_{j} \theta_j^l \right)^2}$$

6. If $b \geq k_i + 1$ and $x_a(k) \geq \frac{m_{a_1}^b + m_{a_2}^b}{2}$

$$\frac{\partial y(k)}{\partial W_c(k)} = \sum_{k} \sum_{j} \left[ \frac{\left( x_a(k) - m_{j_2}^k \right)}{\left( \sigma_j^k \right)^2} \cdot \theta_j \cdot \frac{\partial y_c^k}{\partial W_c} \right] \cdot \sum_{j} \theta_j^i \cdot z_i^j \cdot \frac{1}{\left( \sum_{j} \theta_j^l \right)^2}$$

$$= \sum_{k} \sum_{j} \left[ \frac{\left( x_a(k) - m_{j_1}^k \right)}{\left( \sigma_j^k \right)^2} \cdot \theta_j \cdot \frac{\partial y_c^k}{\partial W_c} \right] \cdot \sum_{j} \theta_j^i \cdot z_i^j \cdot \frac{1}{\left( \sum_{j} \theta_j^l \right)^2}$$

7. If $b \leq k_i$ and $x_a(k) < m_{a_1}^b$

$$\frac{\partial y(k)}{\partial W_c(k)} = \sum_{k} \sum_{j} \left[ \frac{\left( x_a(k) - m_{j_2}^k \right)}{\left( \sigma_j^k \right)^2} \cdot \theta_j \cdot \frac{\partial y_c^k}{\partial W_c} \right] \cdot \sum_{j} \theta_j^i \cdot z_i^j \cdot \frac{1}{\left( \sum_{j} \theta_j^l \right)^2}$$

$$= \sum_{k} \sum_{j} \left[ \frac{\left( x_a(k) - m_{j_1}^k \right)}{\left( \sigma_j^k \right)^2} \cdot \theta_j \cdot \frac{\partial y_c^k}{\partial W_c} \right] \cdot \sum_{j} \theta_j^i \cdot z_i^j \cdot \frac{1}{\left( \sum_{j} \theta_j^l \right)^2}$$

8. If $b \leq k_i$ and $x_a(k) \geq m_{a_1}^b$
\[
\frac{\partial y(k)}{\partial W_c(k)} = \frac{\sum_k \sum_j^m \left[ \frac{x_a(k) - m_{j2}^k}{(\sigma_j^k)^2} \cdot \theta_{r,j} \cdot \frac{\partial y_c^k}{\partial W_c} \right] \cdot \sum_j^{m_j} \theta_{l,j}^{z_l^j}}{(\sum_j^{m_j} \theta_{l,j}^{z_l^j})^2} - \sum_k \sum_j^m \left[ \frac{x_a(k) - m_{j2}^k}{(\sigma_j^k)^2} \cdot \theta_{l,j}^{z_l^j} \cdot \frac{\partial y_c^k}{\partial W_c} \right] \sum_j^{m_j} c_j \cdot \frac{\partial y_c^i}{\partial W_c}
\]

(29)

We can get \( \frac{\partial y_c}{\partial W_c} \), using equation (38) – (55), which we will be further described in section IV.

**IV. T2TSK ALGORITHM FOR IDENTIFICATION**

As described in the preceding discussions, to perform control structure according to Fig. 4, a secondary path model is required. We use T2TSK with standard Back Propagation learning algorithm for this purpose, where the objective is to obtain the secondary path model that represents the dynamics of the true secondary path. Block diagram of the T2TSK for secondary path identification is shown in Fig. 5. The difference between output signal of the true secondary path and a secondary path model represents error of identification process.

![Fig.5. Block Diagram of the T2TSK for secondary path identification](image)
\[ e(k) = y(k) - \hat{y}(k) \] 

(30)

Update of T2TSK parameters in secondary model is formulated by the following equations

\[
W(k + 1) = W(k) - \eta \frac{\partial E(k)}{\partial W(k)} 
\] 

(31)

\[
m^b_{a1}(k + 1) = m^b_{a1}(k) + \eta \frac{\partial E(k)}{\partial m^b_{a1}} 
\]

(32)

\[
m^b_{a2}(k + 1) = m^b_{a2}(k) + \eta \frac{\partial E(k)}{\partial m^b_{a2}} 
\]

(33)

\[
\sigma^b_a(k + 1) = \sigma^b_a(k) + \eta \frac{\partial E(k)}{\partial \sigma^b_a} 
\]

(34)

\[
c^b_a(k + 1) = c^b_a(k) + \eta \frac{\partial E(k)}{\partial c^b_a} 
\]

(35)

\[
s^b_a(k + 1) = s^b_a(k) + \eta \frac{\partial E(k)}{\partial s^b_a} 
\]

(36)

Gradient of the square error in secondary path model is formulated by

\[
\frac{\partial E(k)}{\partial W(k)} = \frac{\partial}{\partial W(k)} \left( \frac{1}{2} (y(k) - \hat{y}(k))^2 \right) 
\]

(37)

\[
= -\frac{1}{2} \cdot 2 \cdot (y(k) - \hat{y}(k)) \frac{\partial \hat{y}(k)}{\partial W(k)} 
\]

\[
= -e(k) \frac{\partial \hat{y}(k)}{\partial W(k)} 
\]

where
\[
\frac{\partial \hat{y}(k)}{\partial c_a^b} = \begin{cases}
\frac{1}{2} \left( \frac{\theta_b^b}{\sum_j^m \theta_l^j} + \frac{\theta_r^b}{\sum_j^m \theta_r^j} \right) \cdot x_a(k) & ; 1 \leq a \leq n \quad (38) \\
\frac{1}{2} \left( \frac{\theta_l^b}{\sum_j^m \theta_l^j} + \frac{\theta_r^b}{\sum_j^m \theta_r^j} \right) & ; a = 0
\end{cases}
\]

\[
\frac{\partial \hat{y}(k)}{\partial s_a^b} = \begin{cases}
\frac{1}{2} \left( \frac{\theta_l^b}{\sum_j^m \theta_l^j} - \frac{\theta_r^b}{\sum_j^m \theta_r^j} \right) \cdot |x_a(k)| & ; 1 \leq a \leq n \quad (39) \\
\frac{1}{2} \left( \frac{\theta_l^b}{\sum_j^m \theta_l^j} - \frac{\theta_r^b}{\sum_j^m \theta_r^j} \right) & ; a = 0
\end{cases}
\]

Because T2TSK structure is developed using Karnik-Mendel algorithm, \( \frac{\partial E(k)}{\partial m_{a1}^b} \), \( \frac{\partial E(k)}{\partial m_{a2}^b} \) and \( \frac{\partial E(k)}{\partial \sigma_a^b} \) can be obtained using the following equations

1. If \( b \geq k_r + 1 \) and \( x_a(k) < m_{a1}^b \)

\[
\frac{\partial \hat{y}(k)}{\partial m_{a1}^b} = \frac{1}{2} \left[ x_a(k) - m_{a1}^a \right] \cdot \frac{\theta_r^b}{\sum_j^m \theta_r^j} \cdot (z_r^b) - c_r
\]

\[
\frac{\partial \hat{y}(k)}{\partial \sigma_a^b} = \frac{1}{2} \left[ \frac{(x_a(k) - m_{a1}^a)^2}{(\sigma_b^a)^3} \right] \cdot \frac{\theta_r^b}{\sum_j^m \theta_r^j} \cdot (z_r^b) - c_r
\]

2. If \( b \geq k_r + 1 \) and \( x_a(k) > m_{a2}^b \)
\[
\frac{\partial \hat{y}(k)}{\partial m_{a2}^b} = \frac{1}{2} \left[ \frac{x_a(k) - m_{b2}^a}{(\sigma_b^a)^2} \right] \cdot \frac{\theta_r^b}{\sum_j^m \theta_r^j} \cdot (Z_r^b) \tag{42} - c_r \\
\frac{\partial \hat{y}(k)}{\partial \sigma_a^b} = \frac{1}{2} \left[ \frac{(x_a(k) - m_{b2}^a)^2}{(\sigma_b^a)^3} \right] \cdot \frac{\theta_r^b}{\sum_j^m \theta_r^j} \cdot (Z_r^b) - c_r \\
\]

3. If \( b \leq k_r \) and \( x_a(k) < \frac{m_{a1}^b + m_{a2}^b}{2} \)

\[
\frac{\partial \hat{y}(k)}{\partial m_{a2}^b} = \frac{1}{2} \left[ \frac{x_a(k) - m_{b2}^a}{(\sigma_b^a)^2} \right] \cdot \frac{\theta_r^b}{\sum_j^m \theta_r^j} \cdot (Z_r^b) \tag{44} - c_r \\
\frac{\partial \hat{y}(k)}{\partial \sigma_a^b} = \frac{1}{2} \left[ \frac{(x_a(k) - m_{b2}^a)^2}{(\sigma_b^a)^3} \right] \cdot \frac{\theta_r^b}{\sum_j^m \theta_r^j} \cdot (Z_r^b) - c_r \\
\]

4. If \( b \leq k_r \) and \( x_a(k) \geq \frac{m_{a1}^b + m_{a2}^b}{2} \)

\[
\frac{\partial \hat{y}(k)}{\partial m_{a1}^b} = \frac{1}{2} \left[ \frac{x_a(k) - m_{b1}^a}{(\sigma_b^a)^2} \right] \cdot \frac{\theta_r^b}{\sum_j^m \theta_r^j} \cdot (Z_r^b) \tag{46} - c_r \\
\]
\[
\frac{\partial \hat{y}(k)}{\partial \sigma_a^b} = \frac{1}{2} \left[ \frac{(x_a(k) - m_{b1}^a)^2}{(\sigma_b^a)^3} \right] \cdot \frac{\theta_r^b}{\sum_j^m \theta_r^j} \cdot \left( Z_r^b \right) - c_r
\]

(47)

5. If \( b \geq k_1 + 1 \) and \( x_a(k) < \frac{m_{a1}^b + m_{a2}^b}{2} \)

\[
\frac{\partial \hat{y}(k)}{\partial m_{a2}^b} = \frac{1}{2} \left[ \frac{x_a(k) - m_{b2}^a}{(\sigma_b^a)^2} \right] \cdot \frac{\theta_i^b}{\sum_j^m \theta_i^j} \cdot \left( Z_i^b \right) - c_i
\]

(48)

\[
\frac{\partial \hat{y}(k)}{\partial \sigma_a^b} = \frac{1}{2} \left[ \frac{(x_a(k) - m_{b2}^a)^2}{(\sigma_b^a)^3} \right] \cdot \frac{\theta_i^b}{\sum_j^m \theta_i^j} \cdot \left( Z_i^b \right) - c_i
\]

(49)

6. If \( b \geq k_1 + 1 \) and \( x_a(k) \geq \frac{m_{a1}^b + m_{a2}^b}{2} \)

\[
\frac{\partial \hat{y}(k)}{\partial m_{a1}^b} = \frac{1}{2} \left[ \frac{x_a(k) - m_{b1}^a}{(\sigma_b^a)^2} \right] \cdot \frac{\theta_i^b}{\sum_j^m \theta_i^j} \cdot \left( Z_i^b \right) - c_i
\]

(50)

\[
\frac{\partial \hat{y}(k)}{\partial \sigma_a^b} = \frac{1}{2} \left[ \frac{(x_a(k) - m_{b1}^a)^2}{(\sigma_b^a)^3} \right] \cdot \frac{\theta_i^b}{\sum_j^m \theta_i^j} \cdot \left( Z_i^b \right) - c_i
\]

(51)

7. If \( b \leq k_1 \) and \( x_a(k) < m_{a1}^b \)}
\[
\frac{\partial \hat{y}(k)}{\partial m_{a1}^b} = \frac{1}{2} \left[ \frac{x_a(k) - m_{b1}^a}{(\sigma_b^a)^2} \right] \cdot \frac{\theta_i^b}{\sum_j \theta_i^b} \cdot (Z_i^b - c_i) 
\]

(52)

\[
\frac{\partial \hat{y}(k)}{\partial \sigma_a^b} = \frac{1}{2} \left[ \frac{(x_a(k) - m_{b1}^a)^2}{(\sigma_b^a)^3} \right] \cdot \frac{\theta_i^b}{\sum_j \theta_i^b} \cdot (Z_i^b - c_i) 
\]

(53)

8. If \( b \leq k \) and \( x_a(k) \geq m_{a1}^b \)

\[
\frac{\partial \hat{y}(k)}{\partial m_{a2}^b} = \frac{1}{2} \left[ \frac{x_a(k) - m_{b2}^a}{(\sigma_b^a)^2} \right] \cdot \frac{\theta_i^b}{\sum_j \theta_i^b} \cdot (Z_i^b - c_i) 
\]

(54)

\[
\frac{\partial \hat{y}(k)}{\partial \sigma_a^b} = \frac{1}{2} \left[ \frac{(x_a(k) - m_{b2}^a)^2}{(\sigma_b^a)^3} \right] \cdot \frac{\theta_i^b}{\sum_j \theta_i^b} \cdot (Z_i^b - c_i) 
\]

(55)

V. SIMULATION RESULTS

As mentioned previously, modeling the ANC secondary path is carried-out prior to control task execution. When applying type-2 TSK Fuzzy that is intended to cancel the noise, we need to have an extension of the Type-1 TSK Fuzzy System [5]. Assuming an interval approximation of the Type-2 TSK Fuzzy, we can use a finite number of type-1 TSK Fuzzy to construct the type-2 TSK Fuzzy system. This means that we can use a finite number of type-1 TSK Fuzzy systems to approximate an adaptive type-2 Fuzzy system. In this simulation study, we decided to use 5 type-1 TSK Fuzzy systems to approximate type-2 TSK Fuzzy system for active noise control. All the remaining parameters are the same as those used in the type-1 case.
The primary and secondary paths are modeled by two linear FIR filters taken from estimation results of primary and secondary paths in ANC measurement test at Laboratory for Control Systems and Computer, ITB. The nonlinearity is represented as saturation functions inserted before primary and secondary paths. Such a saturation function can be used to represent nonlinearity of the ANC actuator. An actuator has typically nonlinear response when it operates with an input signal having an amplitude close to (or above) the nominal input signal value. Block diagram of this simulation is shown in Fig 5.

Results of T2TSK identification are shown in Fig. 6. In this simulation, a sinusoidal signal of 200 Hz (noise frequency) was applied to excite the secondary path. Sampling frequency of 2030 Hz was used throughout the simulation. As shown in Fig 6, the resulting T2TSK model approximates the true ANC secondary path with acceptable accuracy. Error of the modeling process reaches the steady state values in acceptable amount of time. The performance of interval type 2 TSK fuzzy is compared with type 1 TSK Fuzzy and FIR algorithm, as shown in Table I.

Further modeling simulation was carried out by using multi reference signal, which is a sum of three sine waves at the normalized frequencies of 85 Hz, 125 Hz and 250 Hz. The results is shown in Fig. 7. Comparison among T2TSK, T1TSK and FIR algorithm in ANC modeling is shown in Table II.

The simulation study shows that for nonlinear secondary path identification process, interval Type-2 TSK fuzzy algorithm performs better than both FIR algorithm and Type-1 TSK fuzzy algorithm for single frequency excitation noise as well as for superposition noise.
Fig 6. Secondary path identification using T2TSK Fuzzy System
(a) Secondary path actual output and secondary path model output.
(b) Error identification.
(c) Power spectral density between secondary path output and secondary path model output
(d) Error power spectral density

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>MSE</th>
<th>SER</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuzzy TSK T2</td>
<td>$7.8 \times 10^{-3}$</td>
<td>18.795 dB</td>
</tr>
<tr>
<td>Fuzzy TSK T1</td>
<td>$8 \times 10^{-3}$</td>
<td>18.72 dB</td>
</tr>
<tr>
<td>FIR</td>
<td>$7.3 \times 10^{-2}$</td>
<td>9.106 dB</td>
</tr>
</tbody>
</table>
Fig 7. Secondary path identification with excitation signal containing three frequencies (85 Hz, 125 Hz, and 250 Hz)
(a) Secondary path actual output and secondary path model output.
(b) Error identification.
(c) Power spectral density between secondary path output and secondary path model output.
(d) Power spectral density of error.

TABLE II
COMPARISON RESULTS BETWEEN T2TSK, T1TSK AND FIR IN SECONDARY PATH IDENTIFICATION WITH EXCITATION SIGNAL CONTAIN THREE FREQUENCIES (85 Hz, 125 Hz, 250 Hz)

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>MSE</th>
<th>SER</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuzzy TSK T2</td>
<td>0.0037</td>
<td>23.3706 dB</td>
</tr>
<tr>
<td>Fuzzy TSK T1</td>
<td>0.0057</td>
<td>21.4776 dB</td>
</tr>
<tr>
<td>FIR</td>
<td>0.0087</td>
<td>19.7893 dB</td>
</tr>
</tbody>
</table>
Fig 8. ANC simulation results using T2TSK with single signal frequency disturbance
(a) residual signal before and after control is activated
(b) power spectral density of residual signal with and without control.

Table III
Comparison results between T2TSK, T1TSK and FIR in ANC with single signal frequencies disturbance (200 Hz)

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>MSE</th>
<th>Global Attenuation (dB)</th>
<th>Attenuation Level (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>200 (Hz)</td>
</tr>
<tr>
<td>T2TSK</td>
<td>0.0011</td>
<td>9,4268</td>
<td>27,9701</td>
</tr>
<tr>
<td>T1TSK</td>
<td>0.0022</td>
<td>6,3664</td>
<td>27,2595</td>
</tr>
<tr>
<td>FxLMS</td>
<td>0.0023</td>
<td>6,3014</td>
<td>29,7877</td>
</tr>
</tbody>
</table>

Fig 9. ANC simulation results using T2TSK with disturbance containing three signal frequencies
(a) residual signal before and after control is activated
(b) power spectral density of residual signal with and without control.

Table IV
Comparison results between T2TSK, T1TSK and FIR in ANC with disturbance containing three signal frequencies (85 Hz, 125 Hz, 250 Hz)

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>MSE</th>
<th>Global Attenuation</th>
</tr>
</thead>
<tbody>
<tr>
<td>T2TSK</td>
<td>0.0027</td>
<td>4,3033 dB</td>
</tr>
<tr>
<td>T1TSK</td>
<td>0.0028</td>
<td>4,1769 dB</td>
</tr>
<tr>
<td>FxLMS</td>
<td>0.0024</td>
<td>4,8022 dB</td>
</tr>
</tbody>
</table>
By employing the Model T2TSK obtained previously in the identification process, T2TSK learning algorithm was then applied for the controller T2TSK in ANC structure of Fig 2. A tonal noise with frequency of 200 Hz was generated through noise source. The results of active noise control are shown in Fig 8. For comparison purpose, similar simulation study was carried out using T1TSK and FIR algorithm. The results are shown in Table III. Results of this simulation study demonstrate that interval type-2 TSK fuzzy algorithm gives better global attenuation than both type-1 TSK and FIR algorithms.

Further ANC simulation was carried out by using multi reference signal, which is sum of three sine waves at the normalized frequencies of 85 Hz, 125 Hz and 250 Hz. The result is shown in Fig. 9. Comparison with T1TSK and FIR algorithm is shown in Table IV. In this case for several sinusoidal waves, interval type-2 TSK fuzzy algorithm results in less global attenuation to global to primary frequency than FIR algorithm, but gives better performance to frequency harmonics.

However, it should be pointed out that computation type of type-2 TSK Fuzzy is more complex than FIR and type 1 TSK Fuzzy algorithm, so the control process takes 37 times slower than FIR and 5 times than type 1 TSK Fuzzy algorithm. While in learning process, interval type 2 TSK Fuzzy algorithm is 68 times slower than FIR and 4 times slower than type 1 Fuzzy algorithm. Because this computation complexity, type-2 TSK Fuzzy only handling noise in frequency lower than FIR and type-1 TSK Fuzzy. For comparison purpose, the interval type-2 TSK Fuzzy algorithm needs 421 units of memory, while FIR only needs 102 unit memory and type-1 Fuzzy algorithm just needs 250 units memory.

VI. CONCLUSION

In this paper, we have proposed active noise control using interval type 2 TSK adaptive fuzzy systems. New algorithms for T2TSK which are suitable for identification and control tasks in ANC were proposed to model and compensate the effect of nonlinearity which may arise in a number of ANC applications. It was shown in general that Type 2 TSK performed better than its Type-1 version and FxLMS algorithm in Active Noise Control.

REFERENCES


